

# Towards Chromatic Homotopy Theory

– The Landweber Exact Functor Theorem –

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10:30 – 18:30

This Bayerische AG presents a modern formulation of the *Landweber Exact Functor Theorem* (LEFT), which stands at the intersection of algebraic topology (spectra & cohomology theories), algebraic geometry (moduli stacks, elliptic curves), and number theory/arithmetic geometry (formal group laws). The LEFT provides a general understanding of how ordinary cohomology theory, complex  $K$ -theory and Thom cobordism theory are related to isomorphism classes of formal group laws; It gives an explicit means of constructing cohomology theories from formal group laws, and is a step towards a potential description of the homotopy group of spheres via modular forms.

We present the classification of formal group laws by the Lazard ring  $L$ , develop the previous examples of complex oriented cohomology theories and their spectra representability in relation with Thom universal complex cobordism  $MU$ , formulate the moduli classification problem via the stack of formal groups  $\mathcal{M}_{FG}$ , and finally lead to the LEFT:

*Any flat graded  $L$ -module  $\mathcal{F}$  over  $\mathcal{M}_{FG}$  provides a complex-oriented homology theory with spectrum  $E(-) = MU(-) \otimes_L \mathcal{F}$ ; such spectra are characterized among ring spectra by their even periodicity.*

It is expected that speakers will illustrate abstract results with explicit formulations and will rely on examples and geometric motivations to remain accessible to algebraic topologists and geometers.

<p><b>Talk 1</b> <b>Formal Groups: Lazard ring &amp; Classifications</b></p> <p>Functorial definition of formal group laws, their endomorphisms and the Lazard ring; characteristic 0 and <math>p</math> classifications by height; Example of Lubin-Tate formal group laws and elliptic curves.</p> <p style="text-align: right;">~ 60 min.</p>	<p><b>Talk 2</b> <b>Complex-oriented Cohomology Theories, Spectra &amp; <math>MU</math></b></p> <p>Vector bundles in singular cohomology and group laws; spectra and Adams-Brown's representability; Thom complex cobordism <math>MU</math> is the universal complex-oriented cohomology theory and Quillen's theorem.</p> <p style="text-align: right;">~ 60 min.</p>	<p><b>Talk 3</b> <b>The stack moduli of formal groups &amp; <math>MU</math> construction</b></p> <p>The moduli of formal groups, the stack <math>\mathcal{M}_{FG}</math> and Hopf algebroids; strict isomorphisms and smash spectra; the height stratification <math>\{\mathcal{M}_{FG}^{\geq n}\}_n</math>; complex oriented cohomologies and <math>MU</math>-spectra as <math>L</math>-sheaves over <math>\mathcal{M}_{FG}</math>.</p> <p style="text-align: right;">~ 60 min.</p>
<p><b>Talk 4</b> <b>Landweber Exact Functor Theorem &amp; Flatness</b></p> <p>Flat quasi-coherent sheaves over <math>\mathcal{M}_{FG}</math> and regular criterion for modules, flatness property of the height stratification; the LEFT and applications to elliptic curves.</p> <p style="text-align: right;">~ 45 min.</p>	<p><b>Talk 5</b> <b>Landweber Exact Functor Theorem &amp; Periodicity</b></p> <p>Spectra representability, phantom maps and evenly generated cohomology theories; Landweber exact functors, flatness and even periodicity; Example of <math>K</math>-theory and of the BP-spectrum.</p> <p style="text-align: right;">~ 45 min.</p>	

The **Bayerische Kleine Arbeitsgemeinschaft** gathers twice a year and gives the opportunity to young researchers (advanced master students, PhD students and postdocs) to actively work on a common topic of Algebraic or Arithmetic Geometry. Topics and organizers are democratically chosen at the end of each meeting following a list of proposals from the participants. Please refer to the website for programmes and more details. If you want to give a talk or attend please contact the organizers.