

Topological Modular Forms

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Universität Bayreuth
Saturday, 14 December 2019
9:30 – 18:00

The goal of this Bayerische Arbeitsgemeinschaft is to introduce interested (aspiring) Algebraic Geometers and Algebraic Topologists to the spectrum called TMF .

TMF stands for Topological Modular Forms, which are in some sense the “higher version” (in the sense of “Higher Algebra”) of the classical ring of modular forms, which is an interesting object of study in Arithmetic Geometry. There, they arise as generalised functions on the moduli space of elliptic curves.

In the first talk, we will revise some fundamental notions of central importance such as formal group laws and their moduli stack \mathcal{M}_{FG} .

After that, we will spend the second and third talk learning about the algebro-geometric theory of elliptic curves, their moduli problem and its solutions via different kinds of moduli schemes and stacks. In the third talk, we shall encounter modular forms from multiple perspectives, e.g. both as certain functions on the upper-half plane and as sections of the sheaf $\omega^{\otimes k}$ on certain moduli spaces of elliptic curves.

In the fourth talk, the object TMF shall finally be introduced. Its construction is difficult and relies on \mathcal{O}^{top} , a sheaf of E_∞ -ring spectra whose existence relies on a theorem of Goerss-Hopkins-Miller.

Finally, in the fifth talk we will try to compute (some of) the homotopy groups of TMF using tools such as the descent and the Adams-Novikov spectral sequence.

Talk 1 Prelude

Formal group laws and formal groups, stacks and topologies, the moduli stack \mathcal{M}_{FG} , the Landweber Exact Functor Theorem.

~ 60 min

Talk 2 Elliptic Curves

Elliptic Curves via Weierstraß equations and as group schemes, the universal elliptic curve over a scheme (with 2 resp. 3 invertible), the notion of elliptic curves over arbitrary base schemes.

~ 60 min

Talk 3 Moduli Spaces and Modular Forms

Moduli problems of elliptic curves, the moduli stack \mathcal{M}_{ell} , the morphism $\mathcal{M}_{ell} \rightarrow \mathcal{M}_{FG}$ (from the last BAG), modular forms classically as functions on the upper half-plane and as sections of the sheaf $\omega^{\otimes k}$, the ring of modular forms.

~ 60 min

Talk 4 Topological Modular Forms

The presheaf \mathcal{O}^{hom} , $TMF[\frac{1}{6}]$, the theorem of Goerss-Hopkins-Miller, the sheaf \mathcal{O}^{top} which is valued in E_∞ -ring spectra, the definition of TMF as the “global sections” of \mathcal{O}^{top} .

~ 60 min

Talk 5 Computation of $\pi_*(TMF)$ via Spectral Sequences

The descent spectral sequence for TMF , the Adams and Adams-Novikov spectral sequences, a comparison map between spectral sequences, the computation of the 3-local homotopy groups of TMF .

~ 60 min



<https://www.timokeller.name/dokuwiki/doku.php?id=tmf>

The **Bayerische Kleine Arbeitsgemeinschaft** gathers twice a year and gives the opportunity to young researchers (advanced master students, PhD students and postdocs) to actively work on a common topic of Algebraic or Arithmetic Geometry. Topics and organizers are democratically chosen at the end of each meeting following a list of proposals from the participants. Please refer to the website for programmes and more details. If you want to give a talk or attend please contact the organizers.