Brauer groups in arithmetic geometry: exercises 7 on the everywhere locally soluble varieties

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For the following exercises, you may use a CAS.

**Exercise 1** (Selmer cubic). Let $C = V_+(3X^3 + 4Y^3 + 5Z^3) \hookrightarrow \mathbb{P}^2_{\mathbb{Q}}$.

1. Show that $C$ is a smooth projective geometrically integral curve of genus 1 over $\mathbb{Q}$.
2. Find the places of bad reduction of $C/\mathbb{Q}$.
3. Show that $C$ has points everywhere locally.

**Exercise 2** (Lind-Reichardt curve). Let $U = V(2y^2 + 17x^4 - 1) \hookrightarrow \mathbb{A}^2_{\mathbb{Q}}$.

1. Find homogeneous equations for a smooth projective geometrically integral model $C \hookrightarrow \mathbb{P}^3_{\mathbb{Q}}$ of $U/\mathbb{Q}$ (Hint: Write it as an intersection of two quadrics).
2. Find the genus of $C$ (Hint: [Hartshorne, Remark IV.6.4.1]).
3. Find the places of bad reduction of $C/\mathbb{Q}$.
4. Show that $C$ has points everywhere locally.