Brauer groups in arithmetic geometry: exercises 7 on the everywhere locally soluble varieties

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For the following exercises, you may use a CAS.

Exercise 1 (Selmer cubic)**.** Let $C = V_+(3X^3 + 4Y^3 + 5Z^3) \hookrightarrow \mathbf{P}^2_{\mathbf{Q}}$.

1. Show that C is a smooth projective geometrically integral curve of genus 1 over $\mathbf{Q}$.
2. Find the places of bad reduction of $C/\mathbf{Q}$.
3. Show that C has points everywhere locally.

Exercise 2 (Lind-Reichardt curve)**.** Let $U = V(2y^2 + 17x^4 - 1) \hookrightarrow \mathbf{A}^2_{\mathbf{Q}}$.

1. Find homogeneous equations for a smooth projective geometrically integral model $C \hookrightarrow \mathbf{P}^3_{\mathbf{Q}}$ of $U/\mathbf{Q}$ (Hint: Write it as an intersection of two quadrics).
2. Find the genus of C (Hint: [Hartshorne, Remark IV.6.4.1]).
3. Find the places of bad reduction of $C/\mathbf{Q}$.
4. Show that C has points everywhere locally.