

Brauer groups in arithmetic geometry: exercises 6

on examples for Brauer groups of schemes

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Exercise 1 (Brauer group of local fields). Calculate $\mathrm{Br}(X)$ for X the spectrum of the ring of integers of a non-archimedean local field (Hint: Brauer group of the residue field) and as a consequence of a non-archimedean local field.

Exercise 2 (Brauer group of models of global fields). Calculate $\mathrm{Br}(X)$ for X the spectrum of the ring of integers of a number field and X a smooth projective geometrically integral model of a global function field (Hint: Albert-Brauer-Hasse-Noether).

Exercise 3 (Brauer group of curves over algebraically closed fields). Calculate $\mathrm{Br}(X)$ for X a smooth proper connected curve over an algebraically closed field (Hint: Tsen's theorem).

Exercise 4 (example of an element of the Brauer group of a surface¹). Let X/\mathbf{Q} be a smooth projective compactification of the affine surface

$$y^2 + z^2 = (c - x^2)(x^2 - c + 1) \neq 0$$

over \mathbf{Q} with $c \in \mathbf{N}$ and $c \equiv -1 \pmod{4}$. Check that $(c - x^2, -1) \in \mathrm{Br}(\mathbf{Q}(X))$ defines an element of $\mathrm{Br}(X)$ of order 2. (Hint: Generalise 8.2.5 in Poonen's book. We will later study the \mathbf{Q} -rational points of such a surface using the Brauer-Manin obstruction.)

¹COLLIOT-THELENE, JEAN-LOUIS; CORAY, DANIEL; SANSUC, JEAN-JACQUES: *Descente et principe de Hasse pour certaines variétés rationnelles*, Journal für die reine und angewandte Mathematik **320**, 150–191 (1980).